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A new vehicle routing problem with application to pathology laboratory service delivery

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Abstract

Accurate and reliable clinical laboratory testing is an important component of a public health approach to disease management in resource-limited settings, with a safe and reliable transportation network playing an integral role in the delivery of the required services. The collection of specimens from a multitude of specimen collection stations and the subsequent transportation of these specimens to respective laboratories for processing, poses a serious logistical challenge to any pathological testing organisation. The specimen collection process is modelled in this paper as a large tetra-objective *vehicle routing problem* (VRP) which may be used as the basis of a decision support system capable of aiding pathological laboratory services in respect of cost-effective planning, routing and scheduling of a large dedicated fleet of vehicles responsible for the delivery of specimens to laboratories. The model builds on a combination of various well-known variants of the celebrated VRP in the literature, but also exhibits novel features, such as an incompatibility between certain types of specimens and laboratories in terms of testing facilities and capabilities available at the laboratories, the equalisation of specimen testing workload across laboratories, limitations in the rates at which the various laboratories can analyse specimens, and time window constraints within which specimens have to be delivered to laboratories.

Key words: Vehicle routing problem, Pathology laboratory service.

1 Introduction

A consultation held in January 2008 in Maputo, Mozambique served to draw up, in collaboration with the World Health Organisation, the Centre for Disease Control and Prevention, the United States Agency for International Development, the American Society for Clinical Pathology, the Clinton Foundation, the Bill and Melinda Gates Foundation and the Supply Chain Management System [5], documentation containing recommendations

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for health care services. The documentation provides a framework for a tiered, integrated network of pathology laboratories with the aim of strengthening laboratory capacity in resource-limited settings. Amongst several other African states, South Africa took part in the development of this framework (and is also a signatory of the Maputo declaration). The South African *National Health Laboratory Service* (NHLS), in fact, consists of such a tiered laboratory network.

These laboratories are partitioned into four tiers: primary laboratories (tier 1), district laboratories (tier 2), regional laboratories (tier 3) and national laboratories (tier 4). The various laboratories have increasing levels of resources and capabilities available with respect to the processing of test specimens as their tier level increases. The tiered level of a laboratory system and the types of testing performed at each level may vary depending on the population served, physical infrastructure available, the level of service available, water and electricity available, road conditions and the availability of trained technical personnel in-country [5].

The purpose of this paper is to put forward a novel variation on the well-known *vehicle routing problem* (VRP) which may be used as a basis for providing decision support for pathology laboratories in respect of the efficient and effective routing and scheduling of its dedicated fleet of specimen collection vehicles.

An acceptable trade-off between four objectives are pursued in this model formulation, namely to minimise the cost associated with the specimen collection routing schedule, to balance the analysis workload at each laboratory, to minimise the longest time spent by a vehicle on the road and finally to minimise the number of vehicles required to implement the specimen collection routing schedule.

This paper is organised as follows. A brief review is conducted in §2 of the large body of literature on the VRP and its variations, after which the objectives and constraints of our VRP formulation are introduced and motivated in §3. The paper closes in §4 with a number of ideas for possible future work related to the proposed formulation.

2 Literature study

The VRP was first introduced into the operations research literature in a paper by Dantzig and Ramser [7] in 1959 who were concerned with the real-world application of delivering gasoline to gas stations. The first mathematical formulation of the VRP was proposed in the paper and an algorithmic solution approach was suggested for the VRP, formulated simply as the celebrated *Travelling Salesman Problem* (TSP) with the addition of a capacity constraint. The VRP should, in fact, rather be viewed as a combination of the TSP and the well-known *bin packing problem*. The algorithm originally proposed by Dantzig and Ramser was limited to small instances of the problem, but in 1964 Clarke and Wright [4] proposed an efficient greedy heuristic for obtaining good solutions to larger instances of the VRP. While the VRP is a generalisation of the TSP, it is much more difficult to solve than the TSP. Exact algorithms exist for the TSP which routinely solve instances with hundreds or thousands of vertices [1] while the best exact algorithms for the VRP can currently only solve instances with roughly a hundred vertices [3, 9].

Significant research interest has been generated by the VRP over the past fifty years. Toth and Vigo [13] have suggested a classification system for variations on the VRP in terms of:

- the underlying transportation network structure,
- the type of transportation requests,
- the constraints that affect each route individually (intra-route constraints),
- the vehicle fleet composition and their home locations,
- various inter-route constraints, and
- the optimisation objectives.

There are numerous approaches toward solving variations on the VRP, depending on the size of the instance and user requirements. The two main approaches to solving these problems involve the use of exact algorithms or metaheuristics. The favoured exact approach has been column generation, introduced by Desrosiers [8] and usually applied to *VRPs with Time Windows*. In instances of the standard *Capacitated VRPs* it often under-performs, however, and so a *branch-and-cut* approach is usually favoured instead, but this superior approach is still limited to instances with fifty customers or less. In 2006, Fukasawa *et al.* [9] introduced a branch-and-cut-and-price algorithm which proved to be more effective in handling larger instances. Continual improvement in exact algorithmic approaches has been made, with instances of more than 150 customers served by 12 vehicles being solved exactly by Contardo [6].

Heuristic solution approaches are almost as old the problem itself, with Dantzig and Ramser [7] introducing a basic heuristic based on the successive matching of customers by the solution of linear programming relaxations and the removal of fractional solutions by trial and error. Since then, numerous *constructive* and *improvement* heuristics have been developed. More recently, effective *metaheuristics* have also been designed which are powerful enough to solve large instances approximately within seconds, with solutions often achieving to within one percent of the optimal objective function value [13]. The most common metaheurisitcs applied to variations on the VRP are tabu search, ant colony optimisation, particle swarm optimisation and simulated annealing [13].

3 Mathematical modelling

In this study we demonstrate how the specimen collection problem of a tiered pathological laboratory service can be translated into the mathematical formulation of a new type of VRP.

3.1 Set notation and parametric configuration

Let $\mathcal{V}^r = \{1, \ldots, |\mathcal{V}^r|\}$ denote the set of homogeneous vehicles that make up the pathological testing service's specimen collection fleet. It is assumed that this set is large enough to facilitate a feasible specimen collection routing and scheduling solution at a 100% service level, as is required by most health care organisations. The homogeneity of the fleet implies that all the vehicles have the same finite freight capacity C_{max} and autonomy level μ (the maximum assignable route length, measured in units of expected time duration). Let b_k represent the home depot within the set of all depots $\mathcal{V}^b = \{1, \ldots, |\mathcal{V}^b|\}$ of vehicle k, from which it sets out on its collection route and to which it must return upon completion of its delivery tour. Let $G = (\mathcal{V}, \mathcal{E})$ represent an undirected graph with vertex set $\mathcal{V} = \mathcal{V}^e \cup \mathcal{V}^d$, representing the set of all specimen collection stations from which pathological specimens have to be collected for analysis $\mathcal{V}^e$ together with the set $\mathcal{V}^d$ of all laboratories to which specimens may be delivered, and edge set $\mathcal{E}$ representing all road connections between destinations in $\mathcal{V}^e$ and $\mathcal{V}^d$. Every point in $\mathcal{V}$ is assumed to be reachable from every other point. Suppose the edge $(i, j) \in \mathcal{E}$ is expected to be traversed in t_{ij} time units at an associated cost c_{ij}. Every specimen collection point $i \in \mathcal{V}$ (laboratory, respectively) has a time window $[a_i, b_i]$ associated with it during which specimens can be collected (delivered, respectively) as well as a service time S_i associated with handling a batch of pathological specimens there. Denote the set of all pathological specimen types by $\mathcal{V}^c = \{1, \ldots, |\mathcal{V}^c|\}$. Each specimen of type $o \in \mathcal{V}^c$ also has an expiration time τ_o before which it must be processed at a laboratory. Every specimen collection station may potentially require different types of specimen collection based on demographic variability and fluctuating demand. Therefore, let $\mathcal{Q}^i = \{q_1^i, \ldots, q_{|\mathcal{V}_c|}^i\}$ represent the specimen collection requirements at specimen collection station i, where q_o^i denotes the volume of specimens of type $o \in \mathcal{V}^c$ requiring collection. Furthermore, define

$$\alpha_{io} = \left\lceil \frac{q_o^i}{C_{max}} \right\rceil,$$

where C_{max} denotes the cargo capacity of a delivery vehicle. Then α_{io} denotes the number of vehicles that have to be scheduled to visit specimen collection station $i \in \mathcal{V}^e$ for the collection of specimens of type $o \in \mathcal{V}^c$. Similar to each specimen collection point, every laboratory $d \in \mathcal{V}^d$ has varying processing capabilities. We utilise the parameter

$$\delta_{do} = \begin{cases} 1, & \text{if specimen type } o \in \mathcal{V}^c \text{ may be transported to laboratory } d \in \mathcal{V}^d \\ 0, & \text{otherwise} \end{cases}$$

to control the delivery of specimens to capable laboratories which can actually analyse these specimens. The different tiered laboratories have different processing rates associated with them, and the decision variable β_{do} denotes the rate at which laboratory $d \in \mathcal{V}^d$ is able to process specimens of type $o \in \mathcal{V}^c$. A maximum processing capacity is also associated with each specimen type at each laboratory. The parameter γ_{do} represents this maximum processing ability at laboratory $d \in \mathcal{V}^d$ in respect of specimens of type $o \in \mathcal{V}^c$.

3.2 Model formulation

In the model formulation, decision variables are required to keep track of the movement and allocation of vehicles to specimen collection stations and laboratories. The decision variable

$$x_{ijk} = \begin{cases} 1, & \text{if vehicle } k \text{ is scheduled to traverse arc } (i, j) \text{ in } \mathcal{E} \\ 0, & \text{otherwise} \end{cases}$$

monitors the movement of vehicle k, while the decision variable

$$y_{iko} = \begin{cases} 1, & \text{if vehicle } k \text{ is scheduled to collect type } o \text{ specimens from customer } i \\ 0, & \text{otherwise} \end{cases}$$

is required to monitor which vehicle services each specimen collection station and what types of specimens each vehicle should transport, since the specimen collection stations exhibit varied service demand. Finally, the decision variable

$$z_{diko} = \begin{cases} 1, & \text{if vehicle } k \text{ is scheduled to transport specimens of type } o \text{ from} \\ & \text{specimen collection station } i \text{ to laboratory } d \\ 0, & \text{otherwise} \end{cases}$$

is required.

Following the discussion in §1, the aim of our model is to pursue an acceptable trade-off between optimising four objective functions. The first of these objectives is to minimise the costs associated with transportation of all the specimens from the specimen collection stations at which they originate to the appropriate laboratories where they are to be analysed, as is standard in most VRPs. This objective may be formulated as

$$\text{minimise } \sum_{i \in \mathcal{V}} \sum_{j \in \mathcal{V}} \sum_{k \in \mathcal{V}^r} c_{ij} x_{ijk}.$$

Our second objective is to balance as much as possible the workload of laboratories in terms of the time required to analyse specimens, which may be formalised as

$$\text{minimise } \max_d \sum_{i \in \mathcal{V}^e} \sum_{k \in \mathcal{V}^r} \sum_{o \in \mathcal{V}^c} \frac{q_i^o}{\beta_{do}} z_{diko}.$$

The third objective is to balance the workload of the delivery vehicles in terms of their total travel time, which may be expressed as

$$\text{minimise } \max_k \sum_{i \in \mathcal{V}} \sum_{j \in \mathcal{V}} x_{ijk} t_{ij}.$$

Our final objective is to

$$\text{minimise } \sum_{k \in \mathcal{V}^r} \sum_{j \in \mathcal{V}} x_{b_k jk},$$

i.e. to minimise the number of vehicles required for specimen collection at a service level of 100% by reducing the number of trips departing from home depots.

The model includes numerous constraints reflecting the requirements of the tiered pathological testing service in respect of the transportation of pathological specimens. The first such constraint set is

$$M \sum_{i \in \mathcal{V}} x_{ijk} \ \geq \ \sum_{o \in \mathcal{V}^c} y_{jko}, \quad j \in \mathcal{V}^e, \ k \in \mathcal{V}^r,$$

which ensures, if vehicle k is scheduled to collect at least one specimen from specimen collection station j, that vehicle k must traverse an arc of the form (i, j) for some $i \in \mathcal{V}^e$ at some point along its route. Here M is a large number (any number larger than $|\mathcal{V}|$ will do). The flow conservation constraint set

$$\sum_{i \in \mathcal{V}} x_{ijk} - \sum_{\ell \in \mathcal{V}} x_{j\ell k} = 0, \quad j \in \mathcal{V}, \ k \in \mathcal{V}^r$$

states that if vehicle k arrives at location j, then the same vehicle must traverse an arc departing from vertex j, for all $j \in \mathcal{V}$ and all $k \in \mathcal{V}^r$.

Another flow constraint set is required to control the delivery of specimens. In this respect, the constraint set

$$y_{iko} - \sum_{d \in \mathcal{V}^d} z_{diko}\delta_{do} = 0, \quad i \in \mathcal{V}^e, \ k \in \mathcal{V}^r, \ o \in \mathcal{V}^c$$

states that if vehicle $k \in \mathcal{V}^r$ is scheduled to collect specimens of type $o \in \mathcal{V}^c$ from specimen collection station $i \in \mathcal{V}^e$, then the same vehicle must deliver these specimens at exactly one feasible laboratory $d \in \mathcal{V}^d$.

It is safe to assume that pathological testing services utilise numerous depots where vehicles may be stored overnight for security reasons. As mentioned in §3.1, all vehicles are assumed to have home depots associated with them. The constraint set

$$\sum_{i \in \mathcal{V}^e} x_{b_k ik} - \sum_{j \in \mathcal{V}^d} x_{jb_k k} = 0, \quad k \in \mathcal{V}^r$$

ensures that vehicle k begins and ends its route at its home depot $b_k \in \mathcal{V}^b$, for all $k \in \mathcal{V}^r$.

As previously stated, each specimen collection station may require collection of different types of specimens by the vehicles and so the constraint set

$$\sum_{k \in \mathcal{V}^r} y_{iko} = \alpha_{io}, \quad i \in \mathcal{V}^e, \ o \in \mathcal{V}^c$$

is incorporated to ensure that vehicles service specimen collection stations appropriately. Let C_{ik} denote the volume of freight in vehicle $k \in \mathcal{V}^r$ when it leaves facility $i \in \mathcal{V}$. The constraint set

$$C_{ik} + \sum_{o \in \mathcal{V}^c} q_o^j y_{jko} - \sum_{o \in \mathcal{V}^c} q_o^j z_{jiko} - C_{max} \leq (1 - x_{ijk})M', \quad i \in \mathcal{V}, \ j \in \mathcal{V}, \ k \in \mathcal{V}^r,$$

is included to ensure that no vehicle capacity constraint is exceeded. More specifically, this constraint set ensures, if vehicle $k \in \mathcal{V}^r$ travels from facility $i \in \mathcal{V}$ directly to facility $j \in \mathcal{V}$, that its freight contents after leaving facility j (*i.e.* its contents after leaving facility i plus the freight collected at facility j less the content delivered at facility j if it is a laboratory) does not exceed the vehicle capacity C_{max}. Here M' is again a large number.

Let T_{ik} be the expected arrival time of vehicle k at destination $i \in \mathcal{V}^e \cup \mathcal{V}^d$. The constraint set

$$T_{ik} + S_i + t_{ij} - T_{jk} \leq (1 - x_{ijk})M'', \quad i \in \mathcal{V}, \ j \in \mathcal{V}, \ k \in \mathcal{V}^r$$

is included to monitor the arrival time of vehicle k at customer j. This constraint set ensures, if vehicle $k \in \mathcal{V}^r$ travels from location $i \in \mathcal{V}$ to location $j \in \mathcal{V}$, that the time instant at which it starts to service the facility at j is bounded from below by the time instant at which it started servicing the facility at i together with the combined service time duration at i and the travel time from i to j. Here M'' is yet again a large number.

Apart from the multiple problem objectives, the novelty of the problem is illustrated in the next constraint set. Each specimen has a certain time window associated with it during which it remains viable. The time window depends on the specimen type, available storage techniques within the vehicles and specimen collection stations, and the potential use of the specimen in question. The constraint set

$$\tau_o - z_{diko} T_{dk} \geq 0, \quad d \in \mathcal{V}^d, \ i \in \mathcal{V}^e, \ k \in \mathcal{V}^r, \ o \in \mathcal{V}^c$$

is incorporated to ensure, if vehicle k is scheduled to transport specimens of type $o \in \mathcal{V}^c$ from specimen collection station $i \in \mathcal{V}^e$ to laboratory $d \in \mathcal{V}^d$ at a later stage along its route, that the difference between collection time and delivery time of these specimens does not exceed its expiration time τ_o.

It is required that the services provided by pathological testing organisations and the respective laboratories are not twenty-four hour operations, but should take place within acceptable time windows associated with each collection point and laboratory. The constraint set

$$a_i \leq T_{ik} \leq b_i, \quad i \in \mathcal{V}^e \cup \mathcal{V}^d, \ k \in \mathcal{V}^r$$

states that vehicle k may not arrive at a specimen collection station or laboratory outside of its associated time window.

If the planning schedule is determined on a daily basis, it is highly unlikely that vehicles will require refueling before returning to their home depots or that the route times will exceed the maximum assignable times stated in standard labour regulations. The constraint set

$$T_{b_k k} \leq \mu, \quad k \in \mathcal{V}^r$$

may nevertheless be incorporated to ensure that vehicle k does not undertake a route which is expected to take longer to complete than its time autonomy level in cases where the overall scheduling is not one day.

Finally, laboratory $d \in \mathcal{V}^d$ is limited in that there is a maximum capacity γ_{do} associated with the processing of specimens of type $o \in \mathcal{V}^c$. The constraint set

$$\sum_{i \in \mathcal{V}^e} \sum_{k \in \mathcal{V}^r} z_{diko} q_o^i \leq \gamma_{do}, \quad d \in \mathcal{V}^d, \ o \in \mathcal{V}^c$$

ensures that the processing capabilities of laboratory $d \in \mathcal{V}^d$ with respect to specimen type $o \in \mathcal{V}^c$ is not exceeded.

4 Future work

The model proposed in the previous section may serve as a starting point in delivering a real-life applicable solution to pathological testing organisations. There are, however, characteristics which have either been simplified or neglected in our model and which will have to be addressed in future in order to create a more realistic model representation.

One such limitation involves the evolution of data. In our formulation the input data were assumed to be static and known a *priori*, but in a real-life application the model would be more useful if it were able to accommodate a dynamic evolution of data and determine when disturbances in the data evolution are considerable enough to warrant triggering the computation of a new routing solution. *Dynamic* VRP formulations of this nature exist in the literature which are able to deal with information revealed over time concerning either customer demand or location.

The notion that it may be more profitable to outsource certain routes due to the isolated nature of certain specimen collection stations may also be investigated and is referred to as *Routing with Profits and Service Selection* in the literature. This variant was first introduced in conjunction with a TSP and then applied later to a VRP. Applying it specifically to pathological collection stations, it would be beneficial to combine the routing costs and profits into a single objective. This specific problem is referred to as *Profitable Tour Problem* and examples of such formulations appear in [2, 10, 12]. Also related to the possibly isolated locations of certain facilities is the option of collecting specimens from positions that are close enough to specimen collection stations. Formulations accommodating this option are referred to as the *Multi-Vehicle Covering Tour Problem* in the literature. Hachicha [11] developed three heuristics specifically to solve this problem.

Due to the nature of the problem formulation presented in §3, metaheurisitcs will be required to obtain good trade-off solutions to instances of our VRP. The VRP formulation proposed in this paper forms part of a larger, ongoing project on local health care decision support at Stellenbosch University. A comparative study of applying different metaheurisitcs to a real case study instance of our VRP will be performed as part of this ongoing project.

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