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Radio transmission tower placement in cellular telephone communication networks

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Abstract

Mobile telecommunication has become an essential communication channel in the modern world. Network providers are faced with the challenge of providing as many people in as many different areas as possible with network service. Multiple factors have to be taken into account when radio transmitter placement decisions are made. Generally, maximum area terrain surface coverage, as well as fail-safe mutual area coverage by at least two transmitters are of prime importance. This results in a bi-objective facility location problem with the goal of achieving an acceptable trade-off between maximising total area coverage by all transmitters in the network, and maximising areas covered by at least two radio transmitters. The network planning problem for second generation networks can be decomposed into the above-mentioned coverage problem and a subsequent real-time frequency assignment problem. For technical reasons, the frequency assignment and coverage objectives cannot be separated in third and fourth generation networks. The focus of this paper is on the planning of second generation networks. In particular, a suitable framework is proposed for evaluating the effectiveness of a given set of placement locations for a network of radio transmitters with respect to both maximum total and mutual area coverage, taking into account obstruction of the line of sight and the first Fresnel ellipsoid between the transmitter and the receiver, which are required to be unobstructed for effective transmission. This is followed by the formulation of a bi-objective facility location model suitable for use as the basis of a decision support system for identifying high-quality trade-offs between maximising total area coverage and maximising mutual area coverage.

Key words: Facility location, Transmitter location, Wireless network planning.

1 Introduction

Mobile telecommunication has revolutionised the modern world. Smartphones and similar devices are used on a daily basis to communicate through many different types of electronic media. This has sparked a trend, especially among the younger generation, of always

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having to be connected and up-to-date on what is going on, not only in their own lives, but also in the lives of others. Similar trends may be seen among business people, who can now use their smartphones or tablets to complete almost any business transaction. It has, therefore, become an absolute priority for cellular telephone network providers to cover as much area as possible in their service provision. Currently, network providers have the choice of using a combination of second, third or fourth generation networks for their service provision.

Second Generation (2G) networks use the *Time Division Multiple Access* (TDMA) protocol to partition the bandwidth bought by the mobile provider into frequency channels of a specific bandwidth, generally in the order of 200 kHz. These channels are then assigned to receivers, each call having one channel allocated to it. Should there be no available channels when a new call is made, the call is blocked until a channel opens to which the call can be assigned. The allowable time during which a call can be blocked is limited and depends on the policy adopted by the network provider. If a channel does not open up during the allowable blocking time, the call is terminated. Channel assignment to the receiver can be performed in one of two ways: *Fixed Channel Assignment* (FCA) or *Dynamic Channel Assignment* (DCA) [6]. In FCA, each base station only has a limited number of channels allocated to it over the available frequency band. These are then assigned to the receivers as calls are made. In DCA, however, the entire bandwidth is available for use by all transmitters and different assignment policies are in place according to which new calls are treated. The general objective in any channel assignment policy is to minimize the number of blocked calls. The channel assignment is usually the final step of the network planning process, but is an operational task, as opposed to a strategic task, and hence repeated in an online fashion, whereas other planning aspects, such as transmitter placement decisions, are strategic and are performed once-off in an off-line fashion [6].

Apart from 2G networks, network providers may also choose from either *Third Generation* (3G) or *Fourth Generation* (4G) networks, also known as *Long Term Evolution* (LTE) networks. These networks use different bandwidth assignment protocols, are more focused on achieving high data download speeds, and are thus usually established in urban locations, where the demand for high download speeds is ever growing. 2G networks, however, are more focused on voice transmission and as a result more common in semi-urban or rural areas.

The choice of the type of network and the resulting placement of radio transmitters forming the network is of primary importance to network providers, especially when taking into account the prospective growth of smartphone users in Africa. Reed *et al.* [8] state that "the number of smartphone connections will rise from about 79 million at the end of 2012 to 412 million by 2018, according to forecasts by Informa." It is, however, not only the number of new smartphone connections that is expected to achieve such impressive growth. 2G networks and feature phones are expected to remain a key aspect of mobile networks in Sub Saharan Africa where, due to the relatively low *Gross Domestic Product* (GDP), smartphones remain beyond reasonable levels of affordability for a large portion of the population. This will especially be the case in semi-urban and rural areas, where new mobile networks are established [2].

Decision support frameworks for transmitter location problems in cellular telephone networks in the literature are mostly based on single-objective optimisation models. In rare instances where multiple placement objectives are incorporated into the underlying optimisation models, these objectives are usually combined into a single weighted-sum model objective. In non-convex models this practice of combining objectives is known to mask Pareto-optimal solutions. Instead, a bi-objective modelling framework is proposed in this paper for uncovering high-quality trade-off solutions to the radio transmitter location problem.

The paper is organised as follows. A concise review is given in §2 of the literature on facility location models which have been used previously to solve similar network planning problems. In §3, a framework is established for evaluating the effectiveness of a given set of transmitter locations. This framework takes into account the obstruction of the line of sight as well as the first Fresnel ellipsoid between transmitter and receiver, both of which are required for effective transmission. A bi-objective facility location model suitable for use as the basis of a decision support system able to identify high-quality trade-off solutions between maximising total area, and maximising mutual area coverage is put forward in §4. Some preliminary computational results are presented in §5 in order to demonstrate the working of the modelling approach. The paper finally closes with a brief conclusion and ideas for future work in §6.

2 Literature Review

For second generation networks, the network planning problem may be decomposed into two distinct phases: *coverage planning*, which involves antennae placement in order to achieve maximum service coverage, and *capacity planning* which involves frequency assignment planning [1]. The coverage planning problem has generally been modelled using variations on the celebrated set covering problem described in the operations research literature. Amaldi *et al.* [1] describe this problem, known in the context of radio transmitter network planning as the *coverage problem*, as follows: Given an area where service provision has to be guaranteed, determine those locations where the radio transmitters should be placed and specify their configurations such that each point (or user) in the service area receives an adequate signal level.

Two main modelling approaches have been adopted in the literature to solve instances of the coverage problem [1]. The first approach follows a continuous optimisation strategy. A specified number, k (say), of base stations are to be located at any site within the given space which is to be covered, where the antennae co-ordinates are the continuous variables of the problem. This space may exclude certain forbidden areas in which no transmitter placements are allowed. In certain cases, other parameters, such as the antennae orientations and/or the transmission power may also be considered as variables. Amaldi *et al.* [1] claim that the most important element of this type of optimisation model is the propagation prediction model used to estimate the signal intensity at each point in the coverage area. Various functions have been developed over the years for signal estimation, ranging from simple empirical models, such as those developed by Hata [3], to more sophisticated ray tracing methods, such as that discussed by Iskander and Yun [4]. The objective function of the coverage problem is usually determined by some measure of the quality of service, such as the largest minimum signal intensity at any location [1]. Due to the high complexity of typical propagation loss functions, global optimisation techniques are usually employed to tackle these problems, as illustrated by Sherali *et al.* [9].

The second coverage problem modelling approach involves the use of discrete mathematical models. In this approach, a number of test sites or demand nodes representing users of the network have to be identified in the service area. Instead of allowing base stations to be placed at any location in the coverage area, discrete mathematical models restrict the positioning of these base stations to a set of so-called *candidate sites*. In these models, the area covered by each base station is determined *a priori*, generally using a radio wave propagation predictor and taking the surrounding topology and morphology of the terrain into account [6]. The area covered by each candidate site is therefore assumed to be known in such an optimisation model.

Krzanowski and Raper [5] explain that in both the continuous and discrete modelling paradigms, *total cover problems* require the determination of the minimum number of facilities required to meet all the demand. *Partial cover problems* arise in contrast when the number of facilities to be placed is fixed and the locations have to be chosen so as to maximise the demand that can be covered using the limited number of facilities. A further extension of the partial cover problem is the so-called *general cover problem*, in which the objective is to minimise the maximum distance between a facility and the demand points it covers. Mathar and Niessen [6] demonstrate how the coverage problem is an extension of the classical minimum cost set covering problem in the operations research literature.

Due to the large dimensions of the optimisation problems typically involved in radio transmitter facility location planning problems, metaheuristics are often employed as approximate optimisation techniques. Simulated annealing has, for example, been used by Mathar and Niessen [6] in an instance where the complexity of the optimisation problem places an optimal solution out of reach. Krzanowski and Raper [5] instead used a hybrid genetic algorithm designed to take the surrounding geography into account during the site selection process.

3 Measuring Coverage

For an area to be considered covered, an unobstructed line of visibility between the transmitter and receiver should at the very least be achieved. If this direct line of visibility is obstructed, then a situation of *Non Line Of Sight* (NLOS) is said to prevail. Radio wave transmission does, however, not only depend on a clear line of visibility between transmitter and receiver. Radio transmission generates an infinite family of nested ellipsoids called *Fresnel ellipsoids*. These ellipsoids all have both the transmitter and receiver at their foci. For effective transmission, the innermost of this family of ellipsoids, called the *first Fresnel ellipsoid*, should also be unobstructed. If an unobstructed line of visibility exists between a transmitter and receiver, but the first Fresnel ellipsoid is partially obstructed, *Near Line Of Sight* (nLOS) is said to have been achieved, whereas if both the direct line of visibility and the first Fresnel ellipsoid between the transmitter and receiver are unobstructed, then (full) *Line Of Sight* (LOS) is said to have been achieved. These notions are graphically illustrated in Figure 1 (a)–(c).

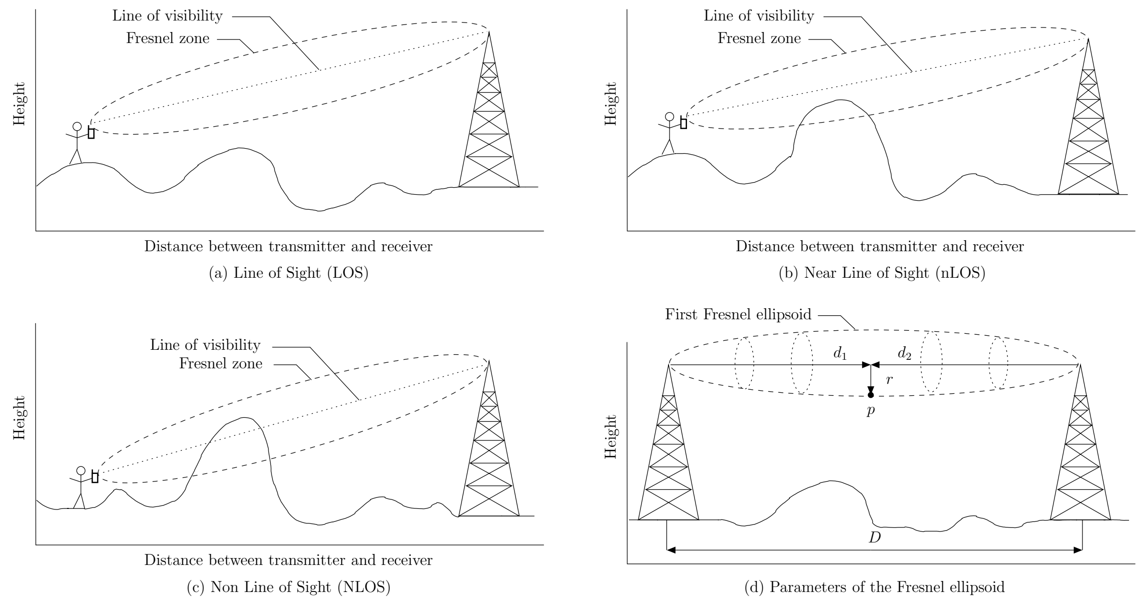

Figure 1: *Various notions related to line of sight and the first Fresnel ellipsoid between a radio transmitter and receiver.*

The radius of the first Fresnel ellipsoid at any point p between the transmitter and receiver is given by

$$r = \sqrt{\frac{\lambda d_1 d_2}{D}}, \tag{1}$$

where d_1 represents the horizontally projected distance between p and the transmitter, d_2 represents the horizontally projected distance between p and the receiver, $D = d_1 + d_2$ is the total distance between the transmitter and receiver, and λ represents the wavelength of the transmitted signal. These parameters are graphically illustrated in Figure 1 (d).

Our decision support framework for radio transmission tower placement is based on a discrete facility location modelling approach, as discussed in §2. The input to the process of determining coverage of an area by a given set of transmitters is a matrix of entries corresponding to a rectangular grid of placement candidate sites (which are also the coverage demand points) containing terrain elevations above sea level for some specified area of interest. For a demand point in this area to be considered covered by a potential transmitter, an unobstructed LOS (*i.e.* an unobstructed line of visibility as well as an unobstructed first Fresnel ellipsoid) has to exist between the transmitter and the demand point. Bresenham's well-known line drawing algorithm, which is widely used in computer graphics to determine which pixels need to be coloured in when drawing straight lines on screen displays, may be used to determine those entries in the matrix which form the line between the transmitter and receiver locations under investigation. Detailed information on Bresenham's line drawing algorithm may be found in [7].

At each of the demand points along the line determined by Bresenham's line drawing algorithm, the difference in elevation between the lower boundary of the first Fresnel ellipsoid and the demand point's elevation above sea level are compared. This is done using the equation of the straight line of visibility between the transmitter and the demand point, and subtracting the radius of the first Fresnel ellipsoid from the height of the line of visibility. The distances d_1, d_2 and D in (1) may be approximated using the theorem of Pythagoras. Only if the elevation of the lower boundary of the first Fresnel ellipsoid between transmitter candidate site i and demand point j is higher than the elevation above sea level for all points along the line determined by the Bresenham line drawing algorithm between i and j, the demand point is considered to be covered by the transmitter candidate site. In this case we populate the entry in row i and column j of a *coverage matrix* with the value $a_{ij} = 1$. Otherwise the value $a_{ij} = 0$ is entered into the coverage matrix.

4 Mathematical Model

Suppose that k transmission towers are to be located at some subset of the transmitter candidate sites, as described in §3. A coverage importance value c_j and a mutual importance value C_j is associated with candidate site $j \in \{1, \ldots, n\}$. The aim of the model is to achieve an acceptable trade-off between maximising the accumulated coverage importance value z of candidate sites actually covered by the k transmission towers and maximising the accumulated mutual importance value Z of those candidate sites that are covered by at least two of the k transmission towers.

We employ the decision variables

$$x_i = \begin{cases} 1 & \text{if a radio transmission tower is placed at site } i \\ 0 & \text{otherwise} \end{cases}$$

for all $i = 1, \ldots, n$ as well as the auxiliary variables

$$y_j = \begin{cases} 1 & \text{if site } j \text{ is covered by at least one transmission tower} \\ 0 & \text{otherwise} \end{cases}$$

and

$$Y_j = \begin{cases} 1 & \text{if site } j \text{ is covered by at least two transmission towers} \\ 0 & \text{otherwise} \end{cases}$$

for all $j = 1, \ldots, n$. The objectives are to

$$\text{maximise } z = \sum_{j=1}^{n} c_j y_j \tag{2}$$

and to

$$\text{maximise } Z = \sum_{j=1}^{n} C_j Y_j \tag{3}$$

subject to the constraints

$$\sum_{i=1}^{n} x_i \leq k \tag{4}$$

$$\sum_{i=1}^{n} a_{ij} x_i \geq y_j, \qquad j = 1, \ldots, n \tag{5}$$

$$\sum_{i=1}^{n} a_{ij} x_i \geq 2Y_j, \qquad j = 1, \ldots, n \tag{6}$$

$$x_i, y_i, Y_i \in \{0, 1\}, \qquad i = 1, \ldots, n. \tag{7}$$

In the above formulation, the objectives in (2) and (3) are conflicting in the sense that increasing z (usually achieved by spacing the transmission towers far apart) typically decreases Z which is, in turn, maximised by placing transmission towers not too far apart. Constraint (4) restricts the number of transmission towers placed to at most k. Constraint sets (5) and (6) are respectively the coverage and mutual coverage requirements. In these linking constraints, the parameter a_{ij} takes the value 1 if the first Fresnel ellipsoid between sites i and j is sufficiently unobstructed, as described in §3. Finally, constraint set (7) enforces the binary nature of the model variables.

5 Model Solution

The framework for determining coverage of a given area as described in §3, as well as the mathematical model of §4, is applied in this section to a real data set containing the elevation data for an area in the Western Cape. The set contains the elevation data for $n = 400$ points forming a 20×20 matrix. The latitude distance between two successive points in the matrix is 308.1 metres, while the longitude distance between two successive points is 256.6 metres. A surface plot of these elevation data is shown in Figure 2.

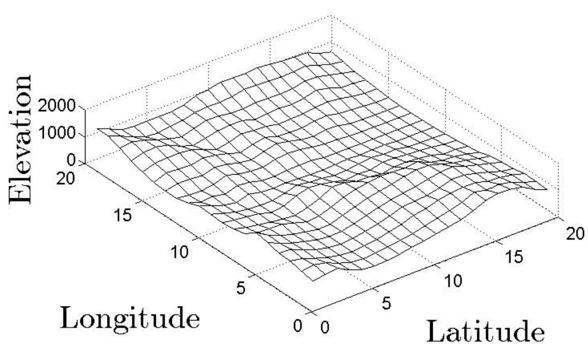

Figure 2: *Surface plot of the elevation data used in the model.*

The frequency used to determine the wavelength required for the calculation of the radii of the first Fresnel ellipsoids is 1 800 MHz, which is commonly used in 2G networks. At this frequency, the wavelength λ is 0.167 metres. The results of the framework for determining coverage are shown in Figure 3 (a)–(b). Figure 3 (a) corresponds to the placement of a transmission tower at latitude position 1 and longitude position 5, while Figure 3 (b) corresponds to the placement of a transmission tower at latitude position 8 and longitude position 1. In these plots, the white area represents the area covered by the transmitter, whereas the black area depicts the parts not covered by the transmitter. For the purpose of demonstration, a road running through the area under consideration is given a single coverage importance value of 1. This importance matrix contains the values of $c_1, \ldots, c_{400}$ in (2) and is shown graphically in Figure 4 (a) where white denotes ones and black denotes zeros. Parts of two towns in the bottom left-hand and top right-hand corners are given a mutual coverage importance rating of 1, as can be seen in Figure 4 (b). The mutual coverage matrix depicted in Figure 4 (b) using the same colour coding scheme contains the values of $C_1, \ldots, C_{400}$ in (3).

The Pareto fronts for this instance of the model (2)–(7) are shown in Figure 5 for the situations in which $k = 2$ and $k = 3$ transmission towers are to be placed. The corresponding locations of the transmitters, together with the percentages of both the single and mutual area coverage achieved by these locations, (measured as the fraction of the areas associated with an importance weighting of 1), are shown in Tables 1 and 2.

Solution	Location 1	Location 2	Single Coverage	Mutual Coverage
1	(10,7)	(11,7)	78.79%	88.06%
2	(1,6)	(9,8)	100%	71.64%
3	(10,7)	(9,8)	93.94%	86.57%
4	(11,7)	(8,9)	96.97%	82.09%

Table 1: *The percentages of the single area as well as mutual area demand coverage achieved by the Pareto-optimal transmission tower placement pairs, located at Location 1 and Location 2.*

It is interesting to note that most of the Pareto-optimal transmitter placements, returned by the model occur in the area ranging from (13,6) to (8,9). This indicates that the best coverage will be achieved by placing a transmitter in that region. Another interesting fact is that in both the cases where $k = 2$ and where $k = 3$, there is a solution covering all the single area coverage demand points which have been given an importance weighting of $c_j = 1$ in (2). In neither case, however, could all the demand for mutual coverage, corresponding to demand points which had been assigned a value $C_j = 1$ in (3) be met. This indicates that in order to provide mutual coverage in those areas, at least four transmitters would have to be placed.

Solution	Location 1	Location 2	Location 3	Single Coverage	Mutual Coverage
1	(13,6)	(12,7)	(8,9)	96.97%	94.03%
2	(13,6)	(8,9)	(17,19)	100%	91.04%
3	(13,6)	(11,7)	(18,20)	69.70%	95.52%

Table 2: *The percentages of the single area as well as mutual area demand coverage achieved by the Pareto-optimal transmission tower placement triples, located at Location 1, Location 2 and Location 3.*

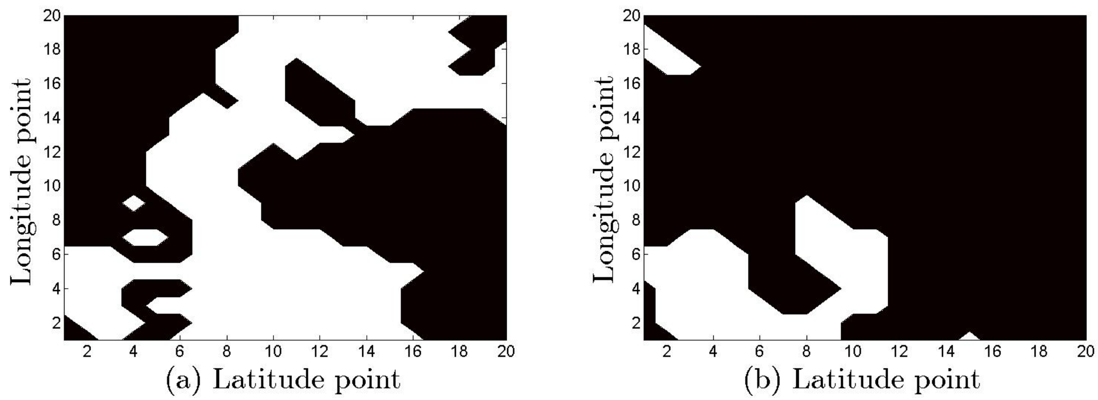

Figure 3: *Viewshed of the area covered by a transmitter placed at (a) point (1,5) and (b) point (8,1).*

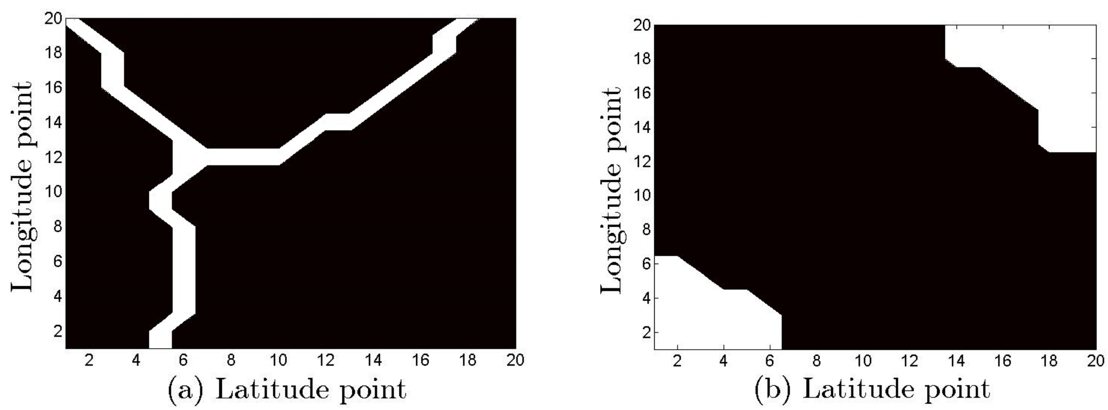

Figure 4: *Contour plot illustrating the importance rating of the area which is to be covered (a) by at least one transmitter and (b) by at least two transmitters.*

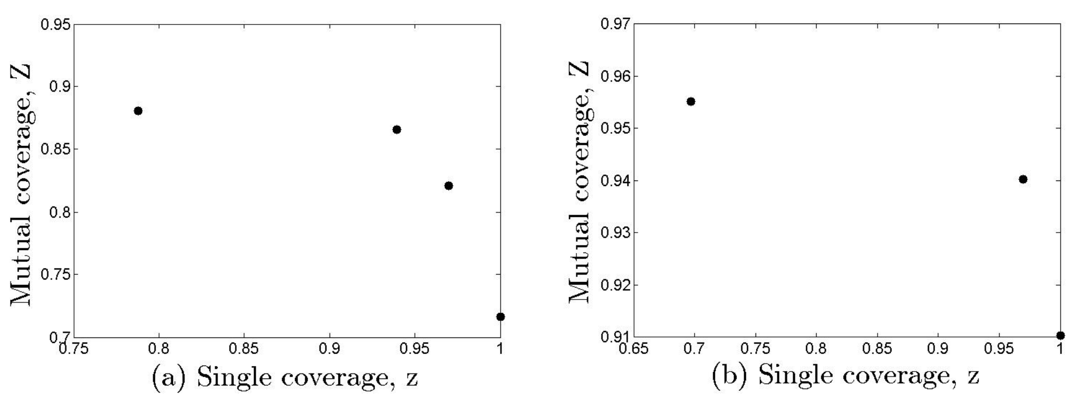

Figure 5: *The Pareto fronts for (a) $k = 2$ and (b) $k = 3$ transmitter placements.*

6 Future Work

The work reported in this paper forms part of a larger, ongoing project on facility location decision support at Stellenbosch University involving various researchers. The next step will be to incorporate radio signal propagation loss into the framework for the evaluation of coverage of a given set of transmitter locations. The propagation loss at a point r incurred from a transmitter located at a point r_0 is defined as the ratio of transmitted power at r_0, $P_t(r_0)$, to the received power at r, $P_t(r)$ [4]. The propagation loss of electromagnetic waves in free space is given by

$$L(dB) = 10 \log \left[\frac{P_t(r_0)}{P_r(r)}\right] = -10 \log \left[\frac{G_t G_r \lambda^2}{(4\pi)^2 D^2}\right], \tag{8}$$

where G_t represents the transmitter gain, G_r represents the receiver gain, λ represents the wavelength of the transmitted signal and D is the distance between the transmitter and the receiver. The propagation loss may not exceed a specified threshold at which the power reaching the receiver will be too small to ensure effective transmission. The expression in (8), however, only allows for the computation of losses along a path loss in free space, ignoring any obstructions by trees, buildings or similar structures which may also have a significant influence on the reduction of the transmitted power. To be able to take this into account, the model developed by Hata and Okumura [3] for path losses in areas where such obstructions may occur is applicable.

The Pareto-optimal solutions reported in §5 were found by brute force (*i.e.* considering all $\binom{400}{2} = 79\,800$ and all $\binom{400}{3} = 10\,586\,800$ location combinations, respectively). As a result, of the high computational complexity, only small instances of the transmitter placement facility location problem can be thus solved. A suitable metaheuristic, such as simulated annealing, will, however, be implemented so that larger instances of the problem can be solved.

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