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Solution representation for a maritime law enforcement response selection problem

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Abstract

Designing a *maritime law enforcement* (MLE) response selection decision support system requires, *inter alia*, an optimization methodology component in which solution search methods are used to provide the decision maker with a set of high-quality solution alternatives to a particular problem instance. In order to facilitate this process, solutions should be encoded in very specific data formats which allow for effective application of local search operations, easy evaluation of objective function values and tests for solution feasibility. The various complex dynamic features associated with this problem, however, make it difficult to standardise these data formats to be used as part of a neighbourhood search process. Consequently, the aim in this paper is to propose an effective solution data encoding scheme that can be incorporated into a real-time MLE response selection decision support system.

1 Introduction

Based on the detection and evaluation of potentially threatening *vessels of interest* (VOIs) at sea, a *maritime law enforcement* (MLE) response selection *decision support system* (DSS) aims to assist human operators in solving the so-called *MLE response selection problem* — allocating and routing of MLE resources, such as patrol vessels, military vessels and armed helicopters, for the purpose of intercepting VOIs.

The proposed DSS consists of a *model generation* subsystem and a *solution search* subsystem. The former subsystem comprises a *model selection* component, which includes the construction and storage of fundamental mathematical structures and fixed modelling components (such as objective functions, routing constraints and MLE resource parameters), and a *model management* component, which consists of dynamic features that are used to model the problem on a temporal basis. The latter subsystem is concerned with finding and presenting a set of non-dominated solutions in multiple objective space to the

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MLE response selection operator for every problem instance formulated in the model generation subsystem. These subsystems share input data *via* a centralised MLE database, as illustrated schematically in Figure 1.

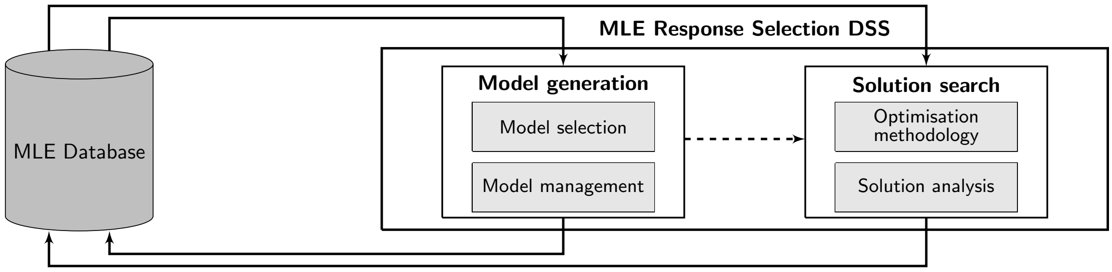

Figure 1: *The MLE response selection DSS with its two subsystems and their components.*

The focus of this paper is on the solution search subsystem of this DSS and the objective is to put forward a flexible solution encoding scheme facilitating the use of a wide range of local search transformation operators and taking into account the crucial end-of-route decisions and other model management aspects associated with the MLE response selection problem.

This paper is structured as follows. Since the MLE response selection problem can be modelled as a multi-depot *vehicle routing problem* (VRP) with a heterogeneous vehicle fleet, a brief literature review on this class of problems is presented in §2, and this is followed by a generic graphical representation of MLE response selection operations in §3. In §4, a discussion is conducted on the end-of-route assignment of MLE resources. Certain features of the model management component, demonstrating the dynamism of the model generation subsystem, are then presented in §5. A method for encoding routing solutions in an MLE response selection DSS is proposed in §6, after which the paper closes with some concluding remarks in §7.

2 Literature review

The multi-depot VRP is known in the literature to be a variant of the capacitated VRP in which routes are simultaneously sought for several vehicles originating from multiple depots, serving a fixed set of customers and then returning to their original depots. Compared to other capacitated VRP variants, a smaller volume of research has been done on the multi-depot VRP, but a number of solution representation techniques have nevertheless appeared in the literature for this problem. Most of these techniques tend to break down an instance into a series of single depot sub-instances and/or only solve it for a single objective and a homogeneous fleet of vehicles [2, 3, 5, 6]. Furthermore, because vehicles always start and finish their routes at the same depots, it is easy to merge the set of customer vertices with that of the depot vertices in such simplified model formulations.

The innovative work of Salhi *et al.* [4], on the other hand, provides a complete mixed integer linear formulation for a generic single-objective multi-depot VRP for a heterogeneous vehicle fleet. In particular, they offer formulation variants for alternative multi-depot

VRP scenarios that are relevant in the MLE response selection routing problem. For instance, they investigate multi-depot VRP variants in which the number of vehicles of a given type is known; some types of vehicles cannot be accommodated at certain depots; or a vehicle is not required to return to the same depot from whence it originated. They then solve the problem using a variable neighbourhood search applied to the notion of *borderline customers*, using six different local search operators.

As described in [1], an MLE response selection problem instance may be modelled as a special type of VRP in which the depots represent the bases from whence MLE resources are dispatched, the fleet of vehicles represents the fleet of MLE resources and the customers represent the VOIs tracked at sea within the territorial waters of the coastal nation. A list of differences between this particular VRP and typical capacitated VRPs encountered in the literature was also given in [1]. The notion of *time stages* was then incorporated into the model formulation in order to accommodate the dynamic nature of the problem: an MLE response selection environment is subjected to so-called *disturbances*, which are threshold phenomena occurring stochastically over time that may cause the currently implemented solution to suffer significantly in terms of quality, hence triggering a new time stage during which the current situation is re-evaluated (*i.e.* the instance is re-solved under the original information combined with the data update which brought along the disturbance).

3 A graphical representation of MLE response selection

A generic graph structure is used to represent the interaction amongst the entities in an MLE response selection environment. The vertex set of this graph is an extension of the vertex set presented in [1], where only the VOIs and MLE resources previously formed part of vertex set for a single depot.

For any given time stage, the vertices in an MLE response selection environment may be partitioned into four sets: VOIs, MLE resources (both active and idle[1]), patrol circuits and bases. While the set of VOIs is typically updated at the start of every time stage due to its high level of dynamism[2], the other three sets remain somewhat more fixed and are updated independently from time stages[3].

Henceforth, let $V^e(\tau) = \{1, \ldots, n(\tau)\}$ represent the set of VOIs at the beginning of time stage τ, let $V^r = \{1, \ldots, m\}$ be the set of MLE resources with respective initial spatial locations $V_0^r(\tau) = \{1_0, \ldots, m_0\}$, let $V^b = \{1, \ldots, |V^b|\}$ denote the set of bases and let $V^p = \{1, \ldots, |V^p|\}$ represent a set of pre-determined patrol circuits. Additionally, let $V(\tau) = V^e(\tau) \cup V^r \cup V^b \cup V^p$. These vertex subsets, along with the arcs inter-linking them, form the directed graph $G(V(\tau), E(\tau))$ depicted in Figure 2, where $E(\tau)$ is the

[1]MLE resources are generally either allocated for the purpose of intercepting VOIs at sea (these MLE resources are then defined as being in a so-called *active* state) or are otherwise strategically allocated to patrol certain areas at sea until needed for law enforcement purposes (these MLE resources are then defined as being in a so-called *idle* state).

[2]Changes in input data linked to a VOI also cause the whole set to be updated.

[3]With occasional exceptions, such as a disturbance caused by an active MLE resource breaking down at sea.

set of pre-calculated arcs linking the vertices and where all pairs of vertices in $V^e(\tau)$ are reachable from one another. Finally, let $V^e_k(\tau) \subseteq V^e(\tau)$ be the set of VOIs scheduled to be investigated by MLE resource k during time stage τ. It is therefore assumed that active MLE resources in V^r are assigned to investigate subsets of VOIs in $V^e(\tau)$ during time stage τ, after which they are assigned to either travel back to a base in V^b or to a designated patrol circuit in V^p.

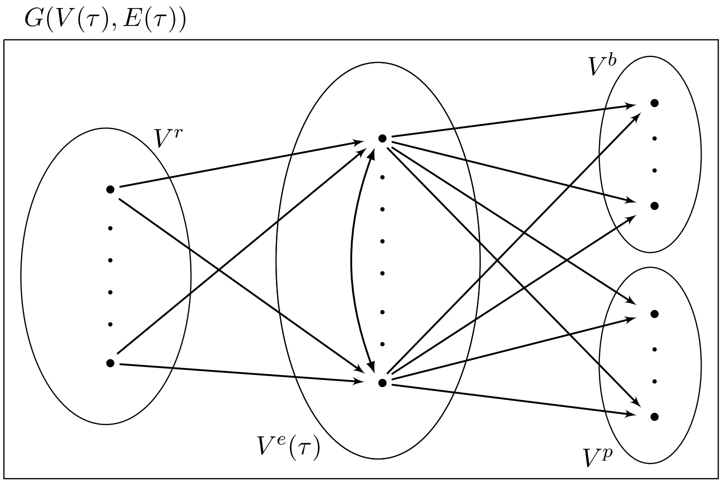

Figure 2: *Graph representation of decisions in an MLE response selection environment.*

4 On end-of-route assignments

This section is devoted to a discussion on the last arc of every route, where active MLE resources transfer from an active state to an idle state in a sub-process defined as *end-of-route* or *post-mission* assignments. This feature of the MLE response selection problem, located in the model management component of the MLE response selection DSS, requires a certain form of input from the idle MLE resources DSS[4]. As discussed in the previous section, after investigating the VOIs assigned to it (*i.e.* completing its mission), the idle MLE resources management operator may either assign an MLE resource to travel to a designated base or to join a designated patrol circuit. Because the MLE response selection process determines where MLE resources will be located in space after completing their missions, it is crucial to consider the impact of this final route arc with respect to operating costs and distance-constrained feasibility.

Due to possible distance-constrained feasibility issues, and because the idle MLE resources management operator cannot know *a priori* where active MLE resources will be located in space after investigating the last VOIs on their routes, so-called *autonomy thresholds* are incorporated into the model formulation. These thresholds ensure that an MLE resource is only allowed to join a patrol circuit after completing its mission provided that the

[4]The DSS in support of the allocation of idle MLE resources in both time and space, which is external to the MLE response selection DSS.

travel distance to the circuit is within a certain autonomy level[5]. Ultimately, a route may only be classified as distance-constrained feasible if there exists at least one approved base that is at most as far away as the autonomy level threshold associated with the MLE resource after having investigated the last VOI on its route. In any case, the idle MLE resources management operator has the power to allocate the resources as soon as they complete their missions, as they then transit into an idle state. Consequently, these idle MLE resources do not have to follow the pre-scheduled end-of-route assignment as dictated by the MLE response selection solution uncovered during the search process at the beginning of the time stage since end-of-route preferences may in the meantime have evolved differently over time.

5 Other aspects of model management

The necessity of establishing a strong format of solution encoding originates from the highly dynamic nature of the MLE response selection problem, where subjective requirements are dictated by the MLE response selection operators on a temporal basis. More specifically, following a disturbance, or simply from a subjective preference point of view, the operators may wish to input additional information to a problem instance in order to accommodate a variety of special requests into the model prior to launching the solution search process for the next time stage. Certain features of the model management component are briefly presented in this section so as to demonstrate examples of dynamic elements introduced during the model generation process. The three features considered in this section are *VOI inclusion sets*, *VOI exclusion sets* and *end-of-route base exclusion sets*.

5.1 Inclusion sets for imposed VOI assignments

In this paper, only the most basic case of inclusion sets is considered, namely *unordered* VOI inclusion sets. These sets are used to force certain VOIs to be intercepted by certain MLE resources, but with no particular degree of urgency in respect of the order in which these VOIs are visited within their routes. Define the unordered inclusion set $I_k(\tau)$ to contain the VOIs forced to be included in the visitation route of MLE resource k during time stage τ. It is assumed that $I_k(\tau) \cap I_\ell(\tau) = \emptyset$ for all $\tau \in \mathbb{N}$ and all $k, \ell \in V^r$, with $k \neq \ell$. Furthermore, $I_k(\tau) \subseteq V_k^e(\tau)$ for all $\tau \in \mathbb{N}$ and $k \in V^r$. To incorporate the above-mentioned visitation requirements into the model formulation in [1], the set of constraints $\sum_{j \in V(\tau) \setminus V^r} x_{ijk}(\tau) = 1,\ i \in I_k(\tau),\ k \in V^r$, may (temporally) be included at the beginning of time stage τ, where $x_{ijk}(\tau)$ is a binary variable which assumes the value 1 if MLE resource k is scheduled to traverse arc (i, j) of the graph in Figure 2 during time stage τ.

[5]The autonomy level of an MLE resource with respect to distance, expressed as a function of time, measures the maximum distance that it may travel at sea before having to return to a designated base.

5.2 Exclusion sets for forbidden VOI assignments

Contrary to inclusion set requirements, the conditions imposed by VOI exclusion sets are met as long as the respective MLE resources are *not* scheduled to intercept VOIs specified in these sets. Define the exclusion set $E_k(\tau)$ to contain the VOIs forbidden to be included in the visitation route of MLE resource k during time stage τ. Then, the set of constraints $\sum_{j \in V(\tau) \setminus V^r} x_{ijk}(\tau) = 0,\ i \in E_k(\tau),\ k \in V^r$, may (temporally) be included in the formulation at the beginning of time stage τ.

5.3 Exclusion sets for forbidden end-of-route base assignments

As discussed in §4, a major dynamic aspect of the MLE response selection problem involves end-of-route assignment decisions, which consists of deciding where certain MLE resources should or should not be sent after completing their missions. For example, the idle management operator may want to control the distribution of idle MLE resources amongst the bases by managing their spread and strategic placement. This information is communicated to the response selection DSS at the beginning of every time stage.

Dictating end-of-route assignments may be achieved in a manner similar to forcing or prohibiting VOI visitations by certain MLE resources. Prohibiting these assignments, for instance, can be achieved by defining $B_k(\tau)$ to contain the bases forbidden to be scheduled for visitation by MLE resource k at the end of its route during time stage τ. Then, the set of constraints $\sum_{i \in V^e(\tau)} x_{ibk}(\tau) = 0,\ b \in B_k(\tau),\ k \in V^r$, may (temporally) be included in the formulation at the beginning of time stage τ.

6 Proposed solution representation scheme

In this section, a suitable method of encoding solutions to the MLE response selection problem is proposed. This encoding scheme is illustrated for a hypothetical problem instance with the following parameters: $V^b = \{B_1, B_2\}$, $V^r = \{a, b, c\}$, $V_0^r(\tau) = \{0_a, 0_b, 0_c\}$, $V^e = \{1, 2, 3, 4, 5, 6, 7\}$ and $V^p = \{P_1, P_2, P_3, P_4\}$.

6.1 Solution strings

In the literature, solutions to a VRP instance are typically encoded as *strings* which comprise substrings representing routes consisting of a subset of customers scheduled to be visited by a particular vehicle. The order in which customers are entered in such a substring is also the order in which the assigned vehicle visits them along its route.

An example of such a solution string for the above hypothetical MLE response selection problem instance is *String 1* of Table 1. In the first route (substring), for instance, MLE resource a is scheduled to first visit VOI 2, then VOI 5, after which it is scheduled to relocate to base B_1. In terms of the decision variables of the combinatorial optimization model proposed in [1], this part of the solution associated with MLE resource a may be written as $x_{0_a 2a} = 1$, $x_{25a} = 1$, $x_{5B_1a} = 1$, and $x_{ija} = 0$ otherwise.

Because the initial and end-of-route cells in the above solution encoding are typically not involved in solution transformations, the string may be simplified before attempting to generate a neighbouring solution during the search process. In particular, the initial and end-of-route cells may be removed and a dummy cell, indicated by the zero element in *String 2* of Table 1 may be placed between routes of different MLE resources.

6.2 String configuration for VOI inclusion and exclusion sets

Part of configuring a solution string involves accommodating the various complexities associated with the dynamic features of the problem and neighbourhood search techniques that may be employed to solve the problem. The use of inclusion sets as proposed in §5.1, for example, imply that any solution transformation resulting in removing one or more VOIs belonging to inclusion sets from their respective routes will generate infeasible neighbouring solutions. One way of eliminating this shortcoming is to remove the VOIs belonging to inclusion sets from the solution string, carry out the solution transformation process with respect to the reduced string, and strategically reinsert these VOIs into feasible substrings[6] after completing the transformation process. Returning to our example, suppose that $I_a(\tau) = \{2\}$, $I_b(\tau) = \emptyset$ and $I_c(\tau) = \{4\}$. The VOIs belonging to any of these sets are then temporally removed from *String 2* to arrive at *String 3* of Table 1.

A random reduced neighbouring string may then be generated from the current reduced string, as shown in *String 4* of Table 1. Here, an inter-route transformation is performed, where VOI 3 and VOI 6 are removed from the second substring (route) and reverse-inserted into the first substring, while VOI 5 is removed from the first substring and inserted into the second substring (a popular neighbourhood move operator in the literature). Following this transformation, the VOIs that were temporally removed are placed back at random positions within their substrings, as shown in *String 5* in Table 1, so as to maintain feasibility.

String 1	$\{\{0_a, 2, 5, B_1\}; \{0_b, 3, 6, B_2\}; \{0_c, 4, 1, P_4\}\}$
String 2	$\{2, 5, 0, 3, 6, 0, 4, 1\}$
String 3	$\{5, 0, 3, 6, 0, 1\}$
String 4	$\{6, 3, 0, 5, 0, 1\}$
String 5	$\{6, 2, 3, 0, 5, 0, 4, 1\}$
String 6	$\{\{0_a, 6, 2, 3, B_2\}; \{0_b, 5, B_1\}; \{0_c, 4, 1, P_3\}\}$

Table 1: *String variants throughout a solution transformation process.*

6.3 String configuration for end-of-route assignments

The next step required to complete the solution string transformation process is to determine where MLE resources are scheduled to end their routes after investigating the VOIs assigned to them. In §4, it was proposed that the idle MLE resource management operator

[6]This does not mean that the generated neighbouring solution is feasible, as there are other routing feasibility criteria that have to be assessed.

must provide some form of input to the MLE response selection operator with respect to end-of-route assignment preferences. Such input may be configured as a set of preferred destinations associated with each active MLE resource, represented as sets containing one or more elements from the patrol circuit and MLE resource base sets. Given the reduced neighbouring solution obtained, as described in §6.2, and the autonomy thresholds, as described in §4, this input may furthermore be filtered by accounting for the positions of the MLE resources after completing their missions[7]. Because the lengths of the last arcs on every route only impact travelling costs in the objective space, the approved (and feasible) end-of-route vertices closest to the last VOI on their respective routes should therefore be configured as the vertices to be visited by the active MLE resources after completing their missions[8].

Let $\overline{B_k}(\tau)$ contain the set of bases that MLE resource k is allowed to travel to after completing its mission during time stage τ, noting that $\overline{B_k}(\tau) \subseteq V^b \backslash B_k(\tau)$. Similarly, let $\overline{P_k}(\tau)$ contain the set of patrol circuits that MLE resource k is allowed to join after its mission during time stage τ. Without loss of generality, it is assumed that an MLE will always be scheduled to join a patrol circuit at the end of its route provided that there is at least such a circuit to join; else it will relocate to one of the approved bases (if possible). Considering our example again, suppose that $\overline{B_a}(\tau) = \{B_2\}$, $\overline{P_a}(\tau) = \emptyset$, $\overline{B_b}(\tau) = \{B_1, B_2\}$, $\overline{P_b}(\tau) = \emptyset$, $\overline{B_c}(\tau) = \{B_1\}$ and $\overline{P_c}(\tau) = \{P_3, P_4\}$. Furthermore, suppose that B_1 is spatially closer to VOI 5 than B_2 is and suppose that P_3 is spatially closer to VOI 1 than P_4 is. Then, after removing the dummy cells and inserting bases or patrol circuits from the sets presented above at the end of their respective routes, the neighbouring string in *String 6* of Table 1 of the solution depicted in *String 1* of the same table results.

7 Conclusion

In this paper, a method of encoding solutions to the MLE response selection problem was proposed. This encoding scheme facilitates independent end-of-route assignments and has been designed for effective use in conjunction with most multi-objective local search techniques. The next step in designing the optimization methodology component is to build a metaheuristic engine that operates, *inter alia*, on the solution encoding scheme proposed (in particular with regard to crossover operators in genetic algorithms), which may be subjected to further adaptations, as well as to develop efficient re-insertion techniques for VOIs that are temporally removed from their routes during the process of generating neighbouring solutions.

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[7] If no such input exists for at least one of the routes, then, clearly, the entire neighbouring solution is classified as infeasible.

[8] The same reasoning cannot be used in the reinsertion process of §6.2, as more than one objectives are simultaneously affected.

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